USN | | | | | | | | | | 10CS34

Third Semester B.E. Degree Examination, June/July 2015
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the following terms and give an example for each :
i) Set ii) proper subset iii) power set iv) nullset. **(04 Marks)**

b. Using Venn diagram, prove that, for any three sets, A, B, C,
i) $\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$
ii) $A - (B \cup C) = (A - B) \cap (A - C)$. **(06 Marks)**

c. In a survey of 120 passengers, an air line found that 48 enjoyed ICE cream with their meals, 78 enjoyed fruits and 66 preferred lime juice. In addition 36 enjoyed any given pair of these and 24 passengers preferred them all. If two passengers are selected at random from the survey sample of 120, what is the probability that :
i) (Event A) they both want only lime juice with their meals?
ii) (Event B) they both enjoy exactly two of the offerings? **(06 Marks)**

d. Prove that the open interval (0, 1) is not a countable set. **(04 Marks)**

2 a. Define a proposition, a tautology and contradiction. Prove that, for any proposition p, q, r, the compound proposition :
$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology. **(07 Marks)**

b. Prove the logical equivalence by using the laws of logic.
$(p \to q) \wedge [\neg q \wedge [r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(07 Marks)**

c. Show that the hypothesis "If you send me an e – mail message, then I will finish writing the program", "If you do not send me an e – mail message, then I will go to sleep early" and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed". **(06 Marks)**

3 a. Define : i) open sentence ii) quantifiers iii) free variable iv) bound variable. **(04 Marks)**

b. Negate the following statement : there exists an integer x such that $2x + 1 = 5$ and $x^2 = 9$. **(06 Marks)**

c. Over the universe of all quadrilaterals in plane geometry, verify the validity of the argument. "Since every square is a rectangle, and every rectangle is a parallelogram, it follows that every square is a parallelogram". **(06 Marks)**

d. Give an indirect proof of the statement "the product of two even integers is an even integer". **(04 Marks)**

4 a. Define : i) Well ordering principle
ii) State and prove that principle of mathematical induction. **(06 Marks)**

b. For all the positive integers n, prove that, if $n \geq 24$, then n can be written as a sum of 5's and / or 7's. **(07 Marks)**

c. If $F_0, F_1, F_2,$ - - - - - are Fibonacci numbers, prove that :

i) $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$ ii) $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(07 Marks)**

PART – B

5 a. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine the following :
i) $|A \times B|$
ii) Number of relations from A to B
iii) Number of binary relations on A
iv) Number of relations from A to B that contain (1, 2) and (1, 5)
v) Number of relation from A to B that contain exactly five ordered pairs
vi) Number of binary relations on A that contain at least seven ordered pairs. **(06 Marks)**

b. Prove that a function $f : A \to B$ is invertible if and only if it is one – to – one and onto. **(07 Marks)**

c. Shirts numbered consecutively from 1 to 20 are worn by 20 students of a class. When any 3 of these students are chosen to be a debating team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any 8 of the 20 are selected, then from these 8 we may form at least two different teams having the same code number. **(07 Marks)**

6 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aR_b if and only if "a is a multiple of b". Write down the relation matrix M(R) and draw its diagraph. **(06 Marks)**

b. For a fixed integer n > 1, prove that the relation "Congruent modulo n" is an equivalence relation on the set of all integers, z. **(07 Marks)**

c. Draw the Hasse diagram representing the positive divisors of 72. **(07 Marks)**

7 a. If $*$ is an operation on z defined by $x * y = x + y + 1$, prove that (z, $*$)is an abelian group. **(06 Marks)**

b. For a group G, prove that the function $f : G \to G$ defined by $f(a) = a^{-1}$ is an isomorphism if and only if G is abelian. **(07 Marks)**

c. State and prove Lagrange's theorem. **(07 Marks)**

8 a. For all x, y, z, $\in z_2^m$, prove that :
i) d(x, y) =d(y, x)
ii) $d(x, y) \geq 0$
iii) d(x, y) = 0 if and only if x = y
iv) $d(x, z) \leq d(x, y) + d(y, z)$. **(06 Marks)**

b. The generator matrix for an encoding function :
$E : Z_2^3 \to Z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

i) Find the code words assigned to 110 and 010
ii) Obtain the associated parity – check matrix
iii) Hence decode the received words : 110110, 111101. **(07 Marks)**

c. Prove that every finite integral domain is a field. **(07 Marks)**

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